spam
ractical analysis

Software for the Practical Analysis of Materials

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ISRD-RCN Workshop:
"Exploring Dynamic Properties of Earth and Planetary Materials Using Neutron and X-Ray Methods"

Grenoble, 23/05/2024

Outline

What is 

?

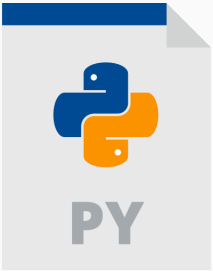
Let's play with some images

General information

What is  ?

What is Spam ?

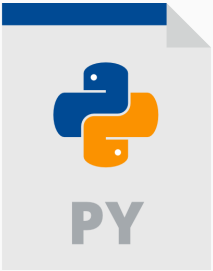
`python` library



(with some bits of `C++`)

What is Spam (practical analysis) ?

`python` library



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data analysis

(not AI enhanced!)

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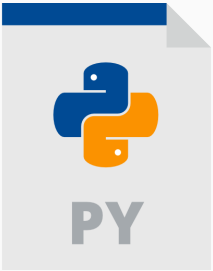

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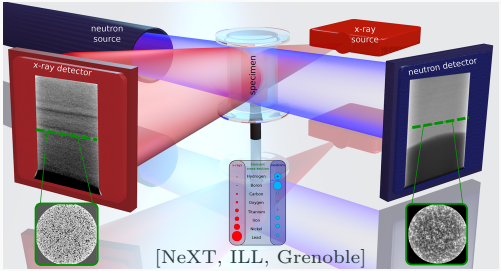
2D images and 3D volumes



[NeXT, ILL, Grenoble]

X-ray & Neutron tomography
(for example)

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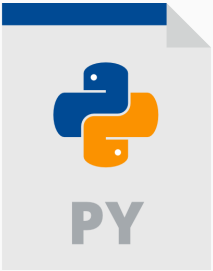

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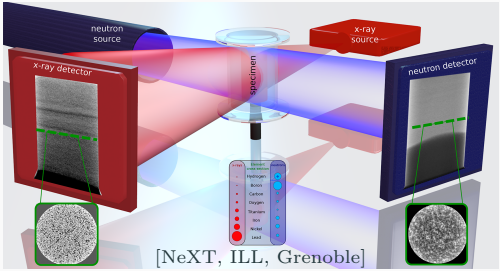
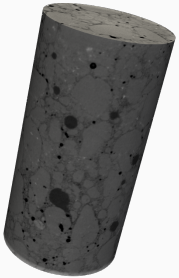


X-ray & Neutron tomography
(for example)

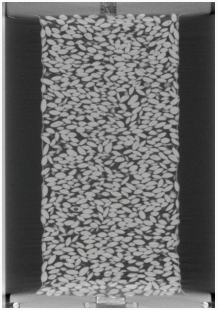
Strictly open-source !

Why do we need a software like Spam ?

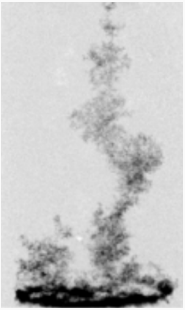
Concrete



Lentils



Sandstone



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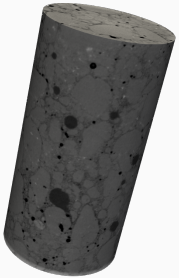

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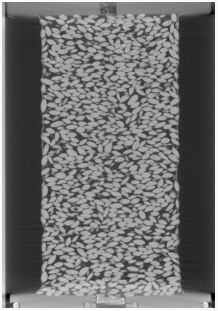

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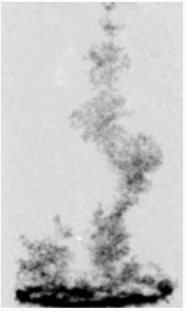

MORPHOLOGY

- Phase identification
- Volume fractions
- Size, shape
- Orientation, fabric
- Tortuosity, connectivity

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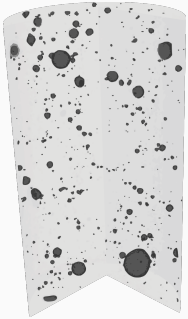

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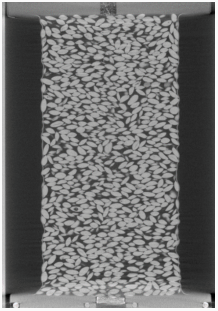

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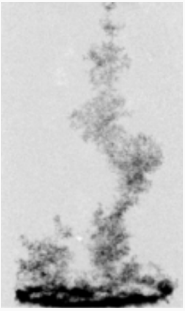

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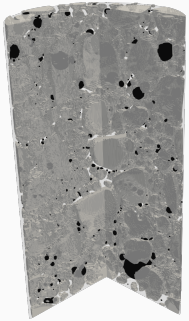

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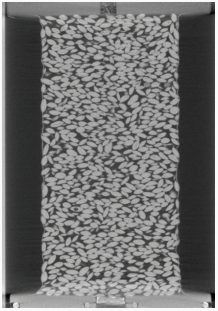

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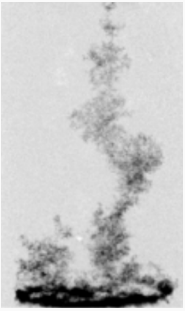

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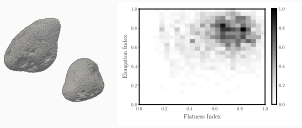
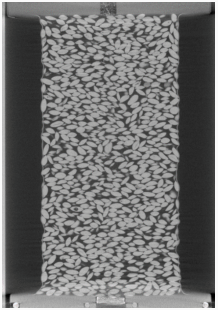


(a) aggregates

(b) macro-pores

Lentils



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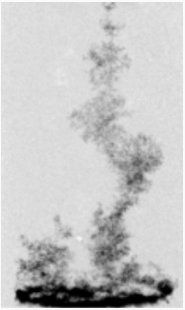

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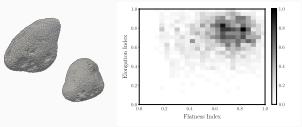

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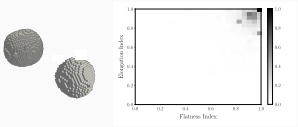

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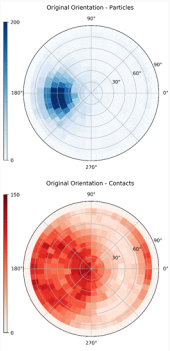

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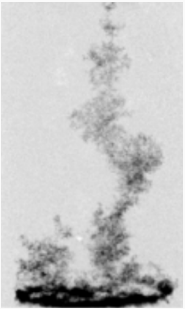

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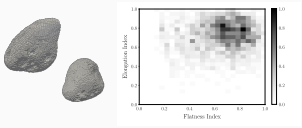

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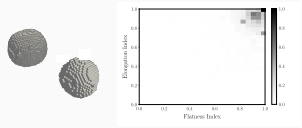

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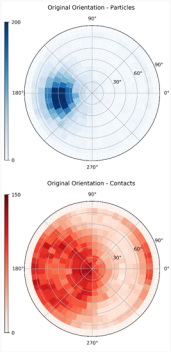

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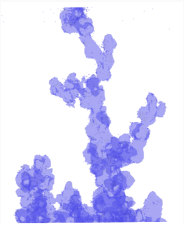

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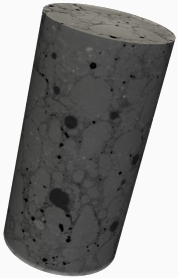

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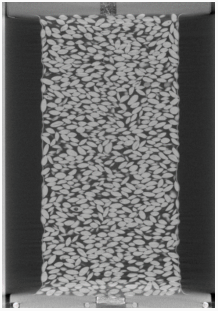

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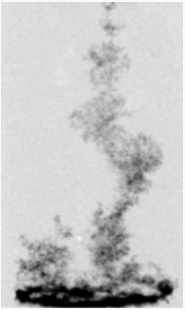
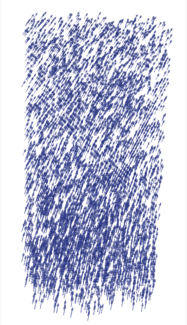


MORPHOLOGICAL EVOLUTION

- δ (Volume fractions)
- δ (Size, shape)
- δ (Orientation, fabric)
- δ (Tortuosity, connectivity)
- KINEMATIC FIELDS

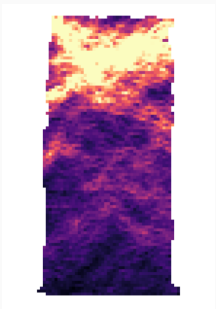
Why do we need a software like Spam practical analysis ?

Concrete displacement field



[Stamati O. (2021) Cem. Concr. Research]

Lentils strain field



[Pinzón G. (2023) Gran. Matter]

Flow in sandstone

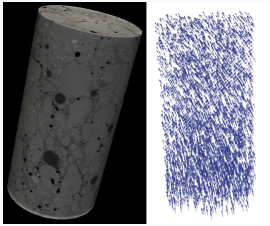
[Couture C. (in preparation)]

MORPHOLOGICAL EVOLUTION

- δ (Volume fractions)
- δ (Size, shape)
- δ (Orientation, fabric)
- δ (Tortuosity, connectivity)
- KINEMATIC FIELDS

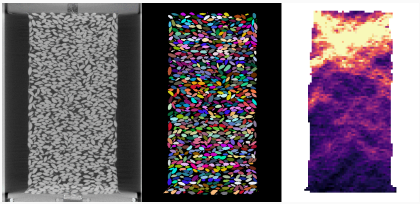
Examples of measuring deforming media with Spam

Concrete fracture



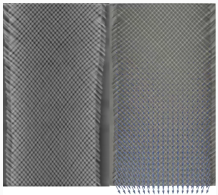
[Stamati O. (2021) Cem. Concr. Res.]

Particle tracking



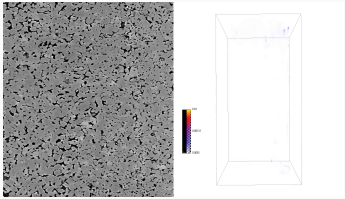
[Pinzón G. (2023) Gran. Matter]

Textile



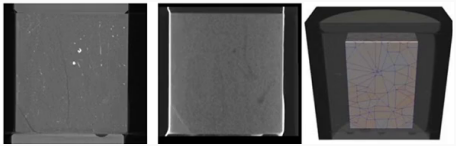
[Stamati O. (2023) Comp. Part A]

Sandstone failure



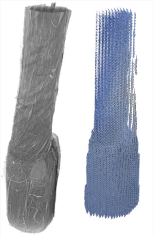
[Cartwright-Taylor A. (2022) Nature Comm.]

Claystone water absorption



[Stavropoulou E. (2020) Front. in Earth Science]

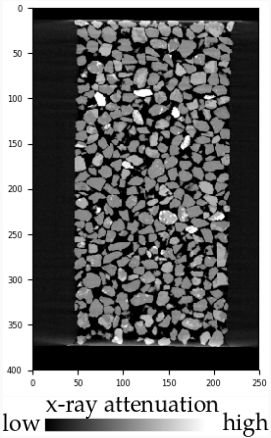
Tendon/Bone Interface



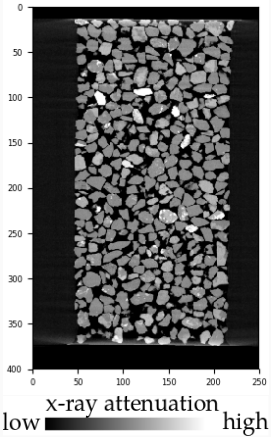
[Sensini A. (in preparation)]

Let's play with some images

Inside an X-ray image



Inside an X-ray image

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```

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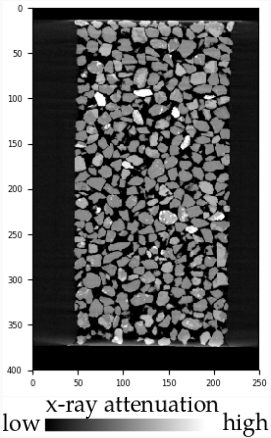


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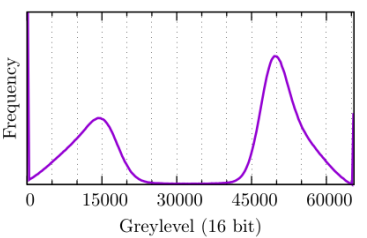

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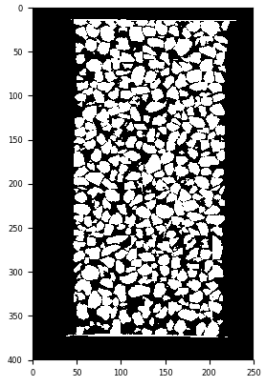

Binarised image

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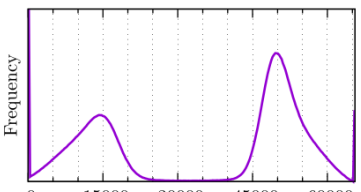
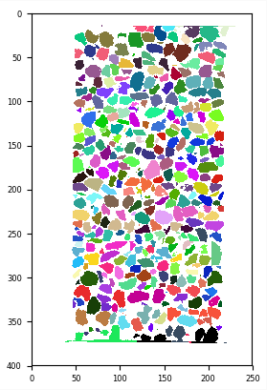


Inside an X-ray image



Labelled image

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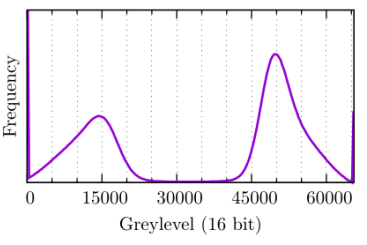
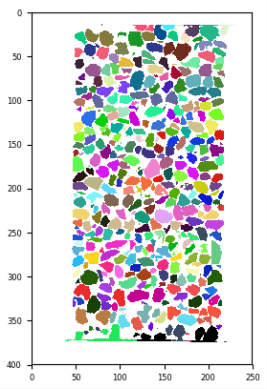


Inside an X-ray image



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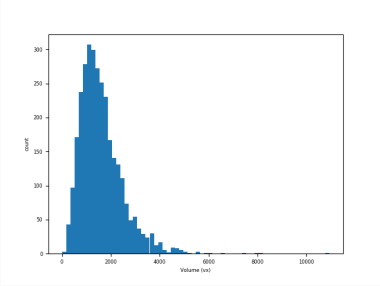

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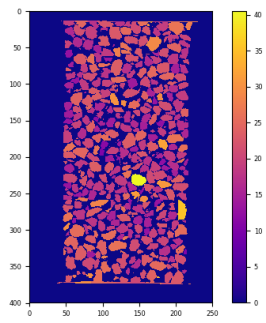

Particle volumes image

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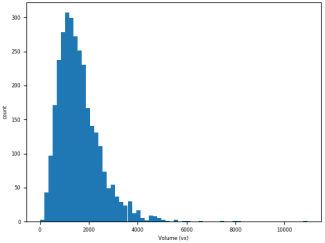
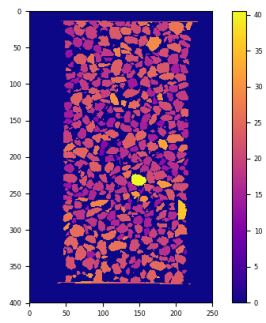


Inside an X-ray image



Particle volumes image

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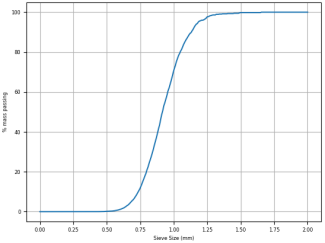
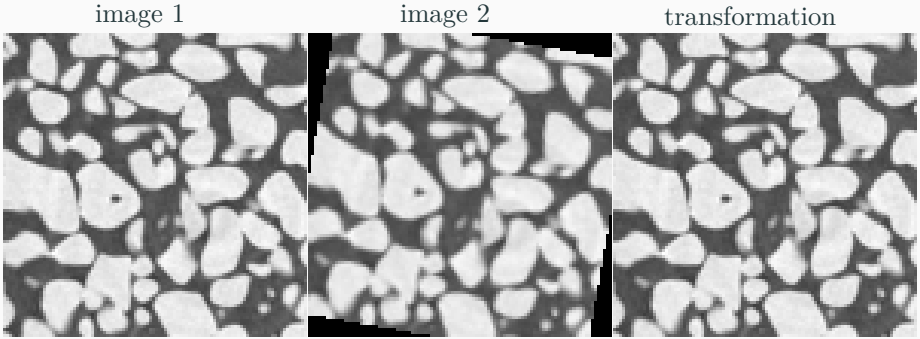


example: Morphological measurements

```python
# import modules
import numpy, tifffile
import spam.label, spam.plotting

# 1. load image and plot the grey level histogram
im = tifffile.imread("grains.tif")
spam.plotting.plotGreyLevelHistogram(grey, showGraph=True)
# 2. identification of grains
binary = im >= 18000
# 3. labelling of grains
labelled = spam.label.watershed(binary)
# 4. volumes calculation
volumes = spam.label.volumes(labelled)
plt.hist(volumes, bins=64); plt.xlabel("Volume (vx)"); plt.ylabel("count"); plt.show()
# 5. plot particle size distribution
radii = spam.label.equivalentRadii(labelled)
spam.plotting.plotParticleSizeDistribution(radii*15*4/1000.0, bins=256, units="mm")
# 6. visualise the grains coloured with volumes
labelledMaxVol = spam.label.convertLabelToFloat(labelled, volumes)
plt.imshow(labelledMaxVol[:, :, labelledMaxVol.shape[2]//2], cmap="plasma"); plt.show()
```

What about the evolution through time?



What about the evolution through time?

image 1

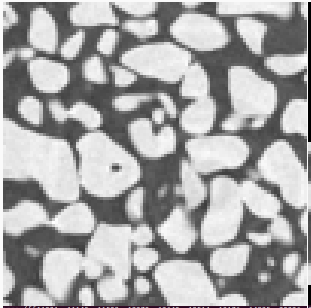


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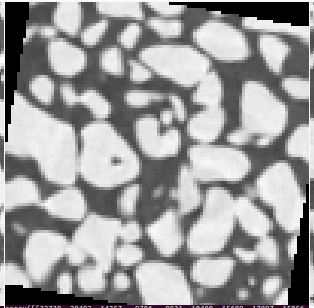
image 2



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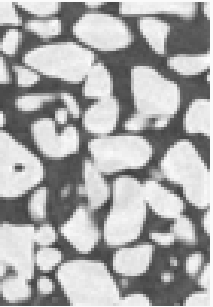
transformation



What about the evolution through time?

image 1

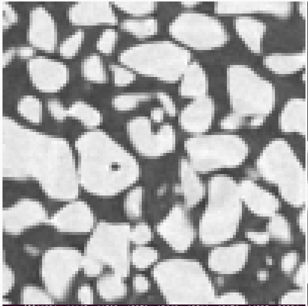


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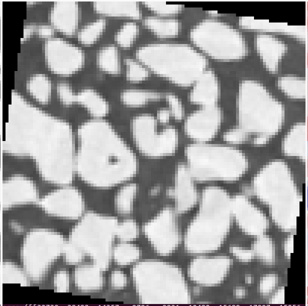
image 2



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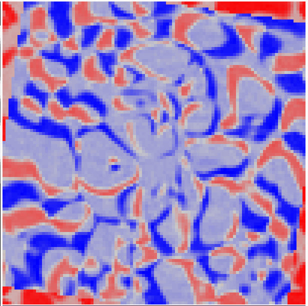
residual



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What about the evolution through time?

image 1

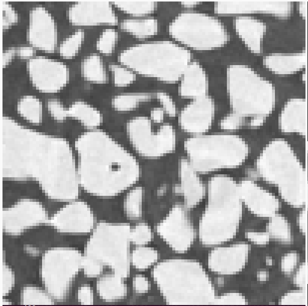

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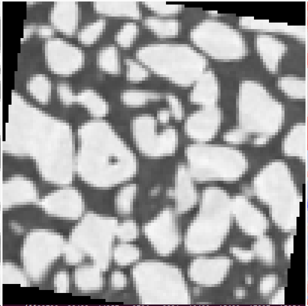
image 2



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       [32045, 26023, 17541, 11582, 11310, 12237, 12162, 12005, 11463,
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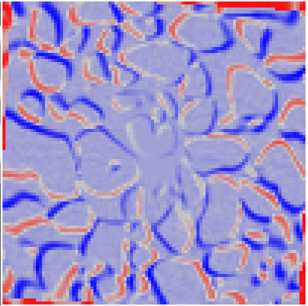
residual



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       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]], dtype=uint16)
```

The general idea of Digital Volume Correlation (DVC)

DVC aims to find a **transformation** that links two 3D images $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$.

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Deformation function in Spam (practical analysis)

$$\boldsymbol{\Phi} = \begin{bmatrix} F_{zz} & F_{zy} & F_{zx} & t_z \\ F_{yz} & F_{yy} & F_{yx} & t_y \\ F_{xz} & F_{xy} & F_{xx} & t_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{with} \quad \boldsymbol{x} = \begin{pmatrix} z \\ y \\ x \\ 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{x}' = \boldsymbol{\Phi} \cdot \boldsymbol{x}$$

where $\boldsymbol{\Phi}$ is **linear** and **homogeneous** accounting for:
translation, **rotation**, **normal** and **shear strain**

Lucas B. D., & Kanade T.: *An iterative image registration technique with an application to stereo vision*, IJCAI Proceedings (1981)

The general idea of Digital Volume Correlation (DVC) in Spam

Gradually **minimise** the SSD between the **reference** image f and the **deformed** image $\widetilde{g}$ corrected by a **trial** deformation function $\widetilde{\boldsymbol{\Phi}}$:

$$\mathcal{T}(\widetilde{\boldsymbol{\Phi}}) = \frac{1}{2} \sum_{\boldsymbol{x} \in \Omega} \left(f(\boldsymbol{x}) - g(\widetilde{\boldsymbol{\Phi}} \cdot \boldsymbol{x}) \right)^2 \quad \text{with} \quad \boldsymbol{\Phi} = \underset{\widetilde{\boldsymbol{\Phi}}}{\operatorname{argmin}}\, \mathcal{T}(\widetilde{\boldsymbol{\Phi}})$$

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A Gauss-Newton iterative scheme is implemented based on a 1st order Taylor expansion of the updated deformed image $\widetilde{g}$ for **small** corrections of $\delta\boldsymbol{\Phi}$:

$$g(\boldsymbol{\Phi}^{(n+1)}\cdot\boldsymbol{x}) = g(\boldsymbol{\Phi}^{(n)}\cdot\boldsymbol{x}) + \boldsymbol{\nabla} g(\boldsymbol{\Phi}^{(n)}\cdot\boldsymbol{x})\cdot\delta\boldsymbol{\Phi}^{(n+1)}\cdot\boldsymbol{\Phi}^{(n)}\cdot\boldsymbol{x}$$

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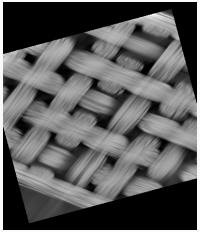
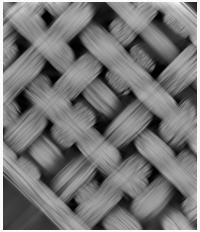
The system in a matrix-vector format:

$$\boldsymbol{M}^{(n)}\delta\boldsymbol{\Phi}^{(n+1)} = \boldsymbol{A}^{(n)}$$

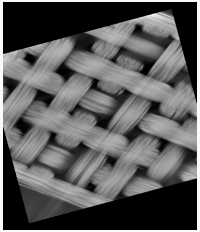
- $\boldsymbol{M}$ is the Hessian: gradient of g and
- $\boldsymbol{A}$ is the Jacobian: contains the difference between f and $\widetilde{g}$



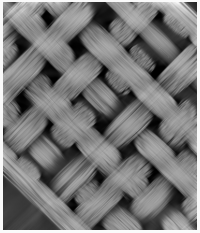
$g(\boldsymbol{x})$



$f(\boldsymbol{x})$

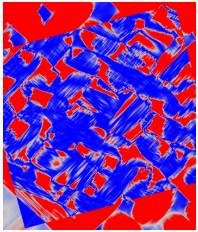


$g(\boldsymbol{x})$

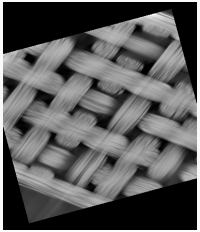


$f(\boldsymbol{x})$

$\|f(\boldsymbol{x}) - g(\boldsymbol{x})\|$

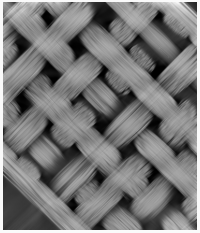


initial difference

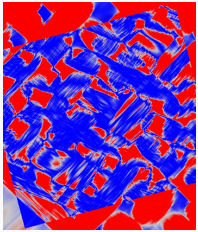


$g(\boldsymbol{x})$

1. Start with initial Φ

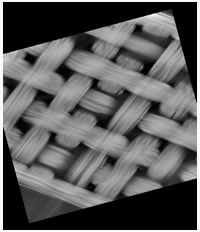


$f(\boldsymbol{x})$

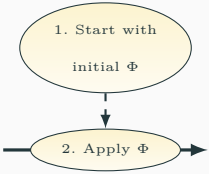


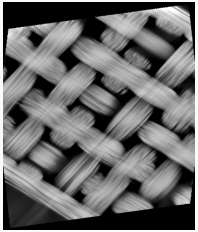
initial difference

Non-rigid registration with Spam

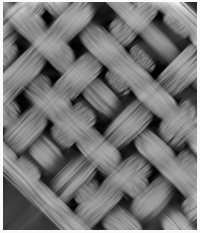


$g(\boldsymbol{x})$

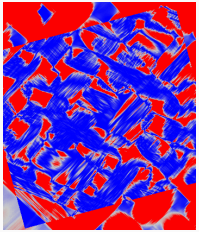




$g(\Phi \cdot \boldsymbol{x})$

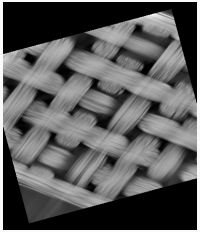


$f(\boldsymbol{x})$



initial difference

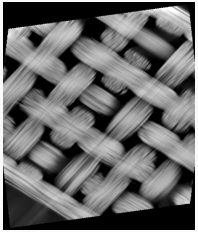
Non-rigid registration with Spam



$g(\boldsymbol{x})$

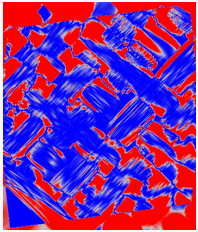
1. Start with initial Φ

2. Apply Φ

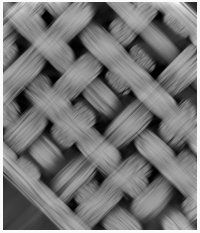


$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$

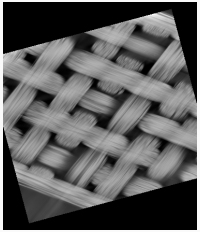


Difference



$f(\boldsymbol{x})$

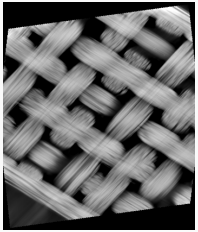
Non-rigid registration with Spam



$g(\boldsymbol{x})$

1. Start with initial Φ

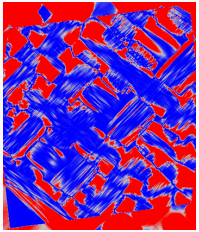
2. Apply Φ



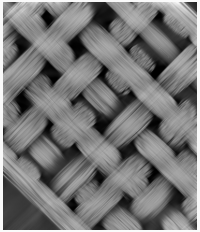
$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$

4. Correction $\delta\Phi$

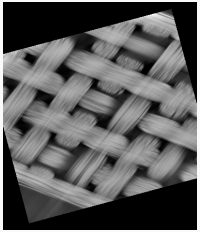


Difference



$f(\boldsymbol{x})$

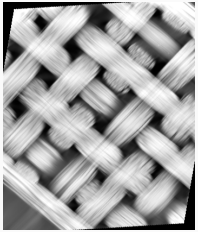
Non-rigid registration with Spam



$g(\boldsymbol{x})$

1. Start with initial Φ

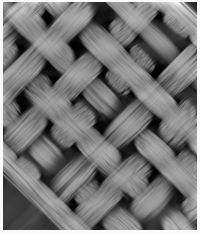
2. Apply Φ



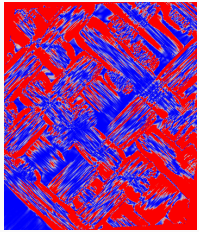
$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$

4. Correction $\delta\Phi$

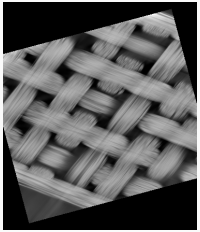


$f(\boldsymbol{x})$



Difference

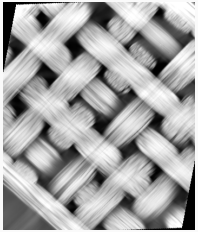
Non-rigid registration with Spam



$g(\boldsymbol{x})$

1. Start with initial Φ

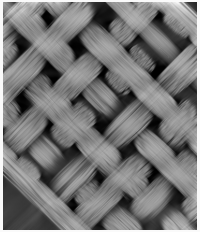
2. Apply Φ



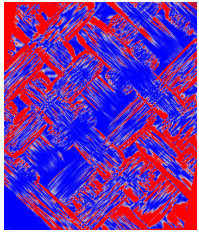
$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$

4. Correction $\delta\Phi$

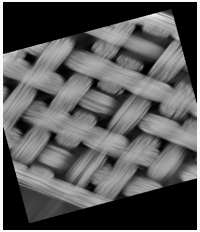


$f(\boldsymbol{x})$



Difference

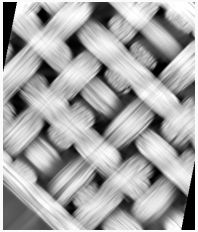
Non-rigid registration with Spam



$g(\boldsymbol{x})$

1. Start with initial Φ

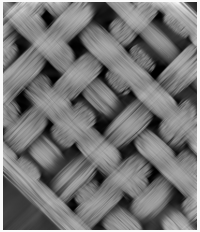
2. Apply Φ



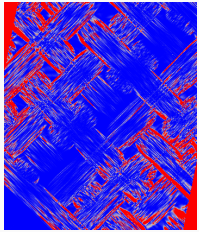
$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$

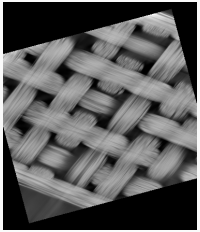
4. Correction $\delta\Phi$



$f(\boldsymbol{x})$



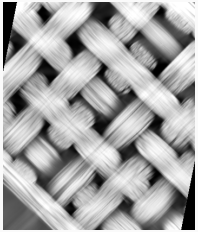
Difference



$g(\boldsymbol{x})$

1. Start with initial Φ

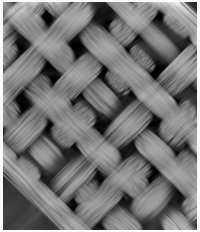
2. Apply Φ



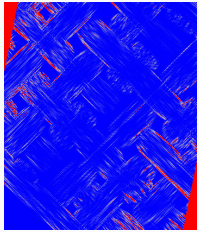
$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$

4. Correction $\delta\Phi$

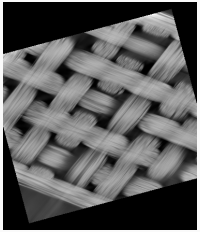


$f(\boldsymbol{x})$



Difference

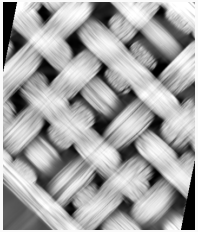
Non-rigid registration with Spam



$g(\boldsymbol{x})$

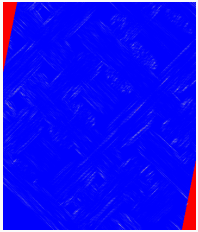
1. Start with initial Φ

2. Apply Φ



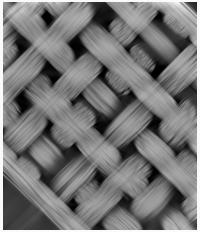
$g(\Phi \cdot \boldsymbol{x})$

3. Calculate $\|f(\boldsymbol{x}) - g(\Phi \cdot \boldsymbol{x})\|$



Difference

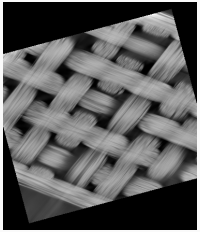
4. Correction $\delta\Phi$



$f(\boldsymbol{x})$

$$\Phi = \begin{pmatrix} 1. & 0. & 0. & 0. \\ 0. & 0.724 & -0.259 & 0. \\ 0. & 0.194 & 0.966 & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

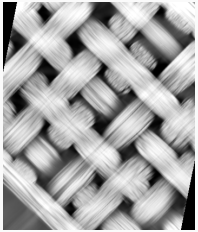
Non-rigid registration with Spam



$g(\boldsymbol{x})$

1. Start with initial Φ

2. Apply Φ



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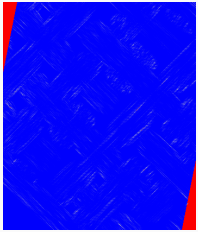

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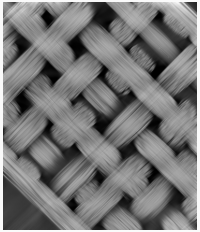

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How to get a deformation field?

$$\mathcal{T}(\widetilde{\mathbf{\Phi}}) = \frac{1}{2} \sum_{\boldsymbol{x} \in \Omega} \left(f(\boldsymbol{x}) - g(\widetilde{\mathbf{\Phi}} \cdot \boldsymbol{x}) \right)^2$$

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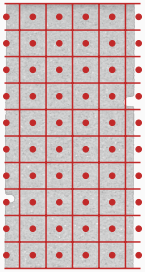

Correlation windows
defined on a regular grid

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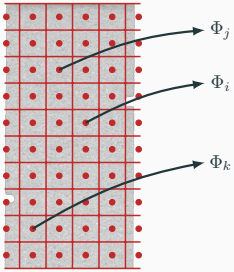

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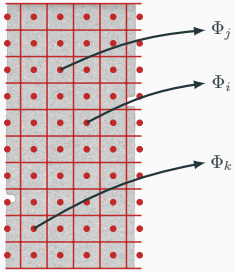

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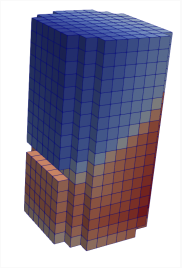
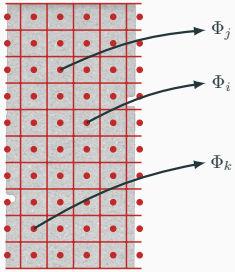


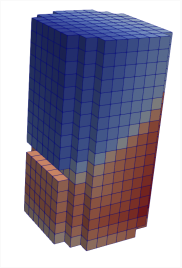
Displacement field

How to get a deformation field?

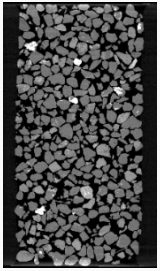
$$\mathcal{T}(\widetilde{\boldsymbol{\Phi}}) = \frac{1}{2} \sum_{\boldsymbol{x} \in \Omega} \left(f(\boldsymbol{x}) - g(\widetilde{\boldsymbol{\Phi}} \cdot \boldsymbol{x}) \right)^2$$



Correlation windows
defined on a regular grid



Displacement field



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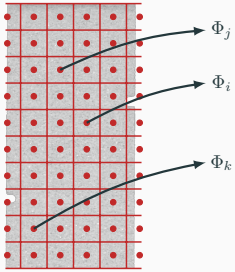
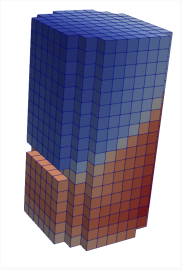
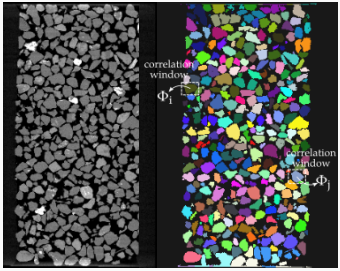


Correlation windows
defined on a regular grid



Displacement field



Correlation windows
defined on labels

Hall S. *et al.*: *Discrete and continuum experimental study of localised deformation in hostun sand under triaxial compression using x-ray ct and 3d digital image correlation*, Géotechnique (2010)

Andò E. *et al.*: *Experimental micromechanics: grain-scale observation of sand deformation*, Géotechnique Letters (2012)

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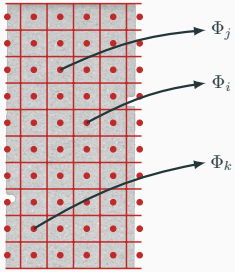
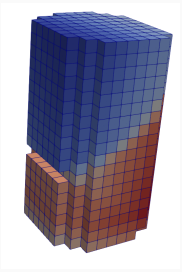
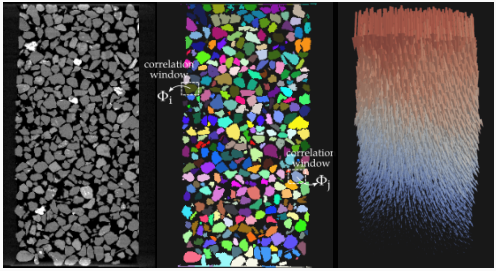


Correlation windows defined on a regular grid



Displacement field



Correlation windows defined on labels

Displacement field

Hall S. *et al.*: *Discrete and continuum experimental study of localised deformation in hostun sand under triaxial compression using x-ray ct and 3d digital image correlation*, Géotechnique (2010)

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How to produce these deformation fields with spam

1. **Initial registration** `script` called from `bash`

```
bash> spam-register                 # The script for the initial alingment
      01.tif 02.tif                 # The two tiff files to load
```

2. **Regular grid DVC** `script` called from `bash`

```
bash> spam-ldic                             # The script for the structured grid
      01.tif 02.tif \                       # The two tiff files to load
      -pf 01-02-registration.tsv \          # Initial guess from the script above
      -hws X \                              # Half-size of the correlation window
      -ns  X \                              # The node spacing of the grid
      -tsv -vtk -tif                        # Ask for TSV, TIFF and VTK file outputs
```

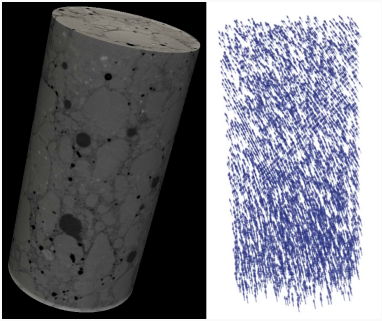
3. **Strains** `script` called from `bash`

```
bash> spam-regularStrains                   # The script for the structured strains
      01-02-ldic.tsv \                      # Deformation field from the script above
      -tsv -vtk -tif                        # Ask for TSV, TIFF and VTK file outputs
```

Read the Docs https://www.spam-project.dev/docs/scripts.html#image-correlation-script-flowchart

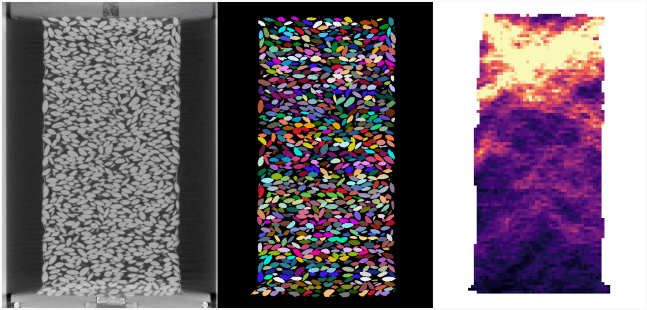
How to get a deformation field through time?

DVC on a regular grid



[Stamati O. (2021) Cem. Concr. Res.]

DVC on labelled objects



[Pinzón G. (2023) Gran. Matter]

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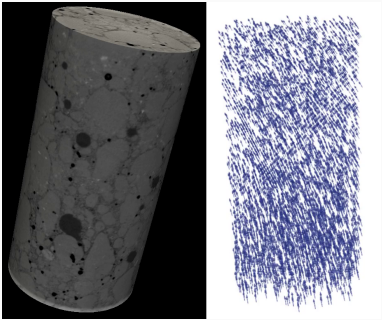
DVC on a regular grid



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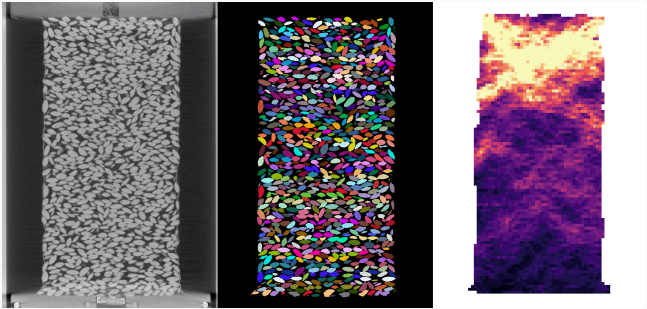
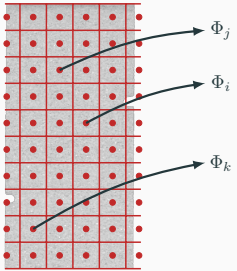


[Pinzón G. (2023) Gran. Matter]

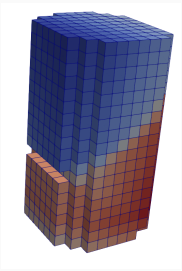
Both are considered as local correlation approaches

Global vs Local DVC approaches

The paradigm behind the global correlation is to transform an image with a **displacement field** which support is a Finite Element mesh instead of a set of linear operators Φ.



Regular grid



Displacement field

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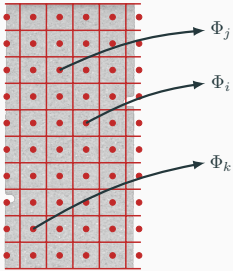
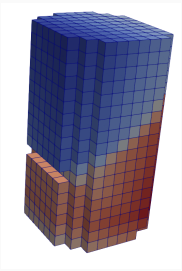
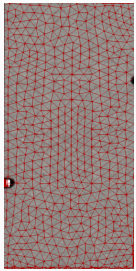


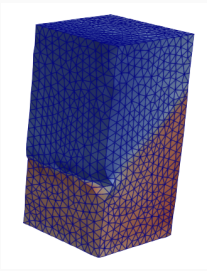
Regular grid



Displacement field



FE mesh



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$$\Phi_{\mathrm{C}} = \sum_{\boldsymbol{x}\in\Omega}\frac{1}{2}\|g(\boldsymbol{x}+\boldsymbol{u}(\boldsymbol{x})) - f(\boldsymbol{x})\|_2$$

Hild F. and Roux S.: *Comparison of local and global approaches to digital image correlation*, Experimental Mechanics (2012)

A Finite Element mesh to transform an image

The global DVC problem can be expressed as the following functional to minimize:

$$\Phi_{\mathrm{c}} = \sum_{\Omega} \|g(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - f(\boldsymbol{x})\|_2$$

where:

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$$\boldsymbol{M}_c \delta \boldsymbol{d} = \boldsymbol{b}$$

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where:

- $\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{N}(\boldsymbol{x})\boldsymbol{d}$ is the displacement field supported but standard FE shape functions
- $\delta \boldsymbol{d}$ is the increment of displacement field unknowns $\boldsymbol{d}$
- $\boldsymbol{M}_c$ can be seen as a FE assembled mass matrix
- $\boldsymbol{b}$ contains the residual supported by the shape functions

Mechanical regularization

Mechanically driven Tikhonov regularization

Equilibrium gap method: Minimizing the distance between current solution $\boldsymbol{u}$ and that which satisfies the **equation for linear elasticity**.

$$\begin{cases} \Phi_c(\boldsymbol{d}) = \sum_\Omega \|g(\boldsymbol{x}+\boldsymbol{u}(\boldsymbol{x})) - f(\boldsymbol{x})\|_2 \end{cases}$$

The regularised problem becomes:

$$(\boldsymbol{M}_c + \boldsymbol{M}_{\text{reg}})\,\delta\boldsymbol{d} = \boldsymbol{b} - \boldsymbol{M}_{\text{reg}}\boldsymbol{d}$$

Mendoza A., Neggers J., Hild F., Roux S.: *Complete mechanical regularization applied to digital image and volume correlation*, Computer Methods in Applied Mechanics and Engineering (2019)

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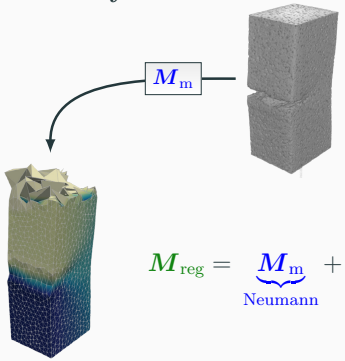

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$$\boldsymbol{M}_{\mathrm{reg}} = \underbrace{\boldsymbol{M}_{\mathrm{m}}}_{\text{Neumann}} + \cdots$$

Mendoza A., Neggers J., Hild F., Roux S.: *Complete mechanical regularization applied to digital image and volume correlation*, Computer Methods in Applied Mechanics and Engineering (2019)

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The regularised problem becomes:

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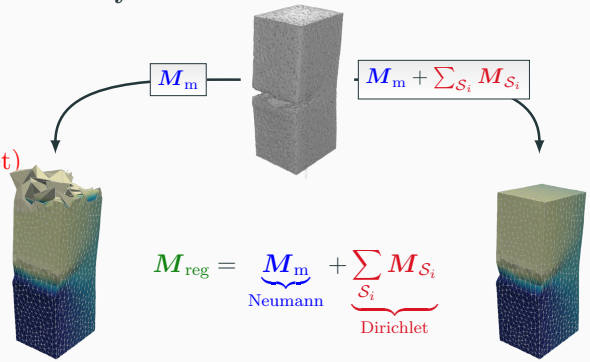
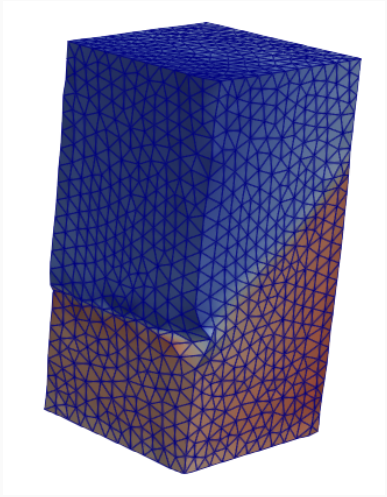


Mendoza A., Neggers J., Hild F., Roux S.: *Complete mechanical regularization applied to digital image and volume correlation*, Computer Methods in Applied Mechanics and Engineering (2019)

Regularized Global DVC



Advantages

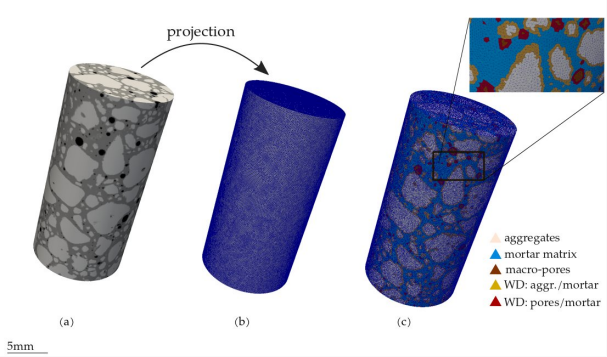
- Continuity of sought displacement field
- Natural regularization
- Mechanically admissible solution
- Straightforward link with simulations
- Can work for non-elastic problems

Global DVC in Spam

- Recently implemented
- Looking for nice data to test it!

https://www.spam-project.dev/docs/spam.DIC.html#module-spam.DIC.globalDVC

Bridging the gap between images and FE models



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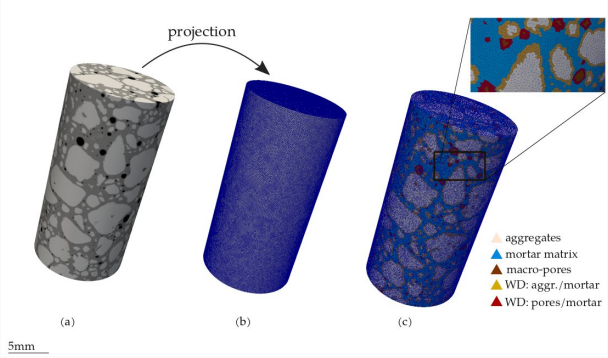
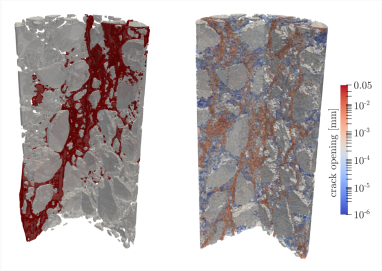
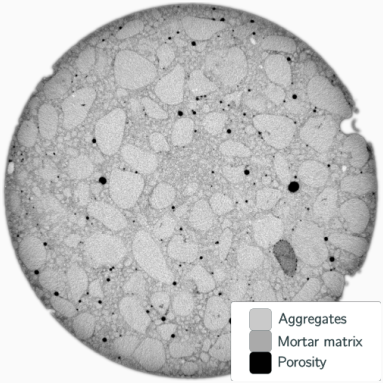




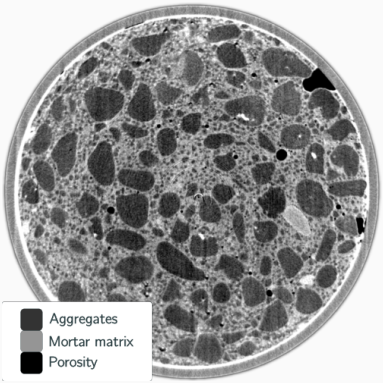
Stamati O., Roubin E., Andò E., Malecot Y.: *Tensile failure of micro-concrete: from mechanical tests to fe meso-model with the help of x-ray tomography*, Meccanica (2019)

https://ttk.gricad-pages.univ-grenoble-alpes.fr/spam/tutorials/tutorial-06-projection.html

Multiple modalities

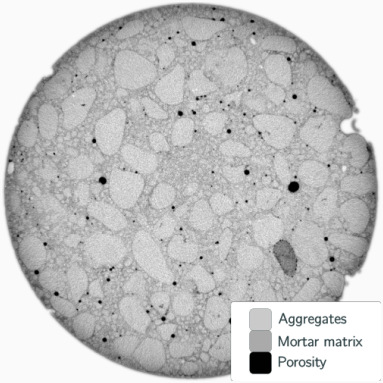


X-ray scan

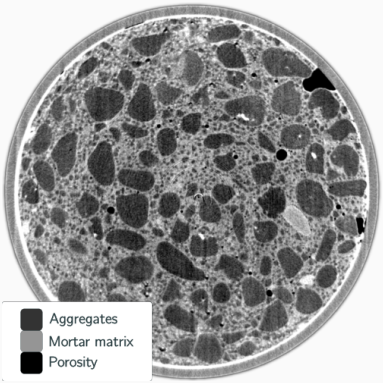


Neutrons scan from D50 (NeXT, ILL)

Multiple modalities



X-ray scan



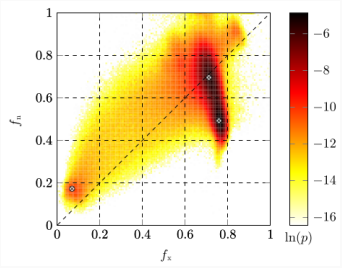
Neutrons scan from D50 (NeXT, ILL)

$$\mathcal{T}(\widetilde{\mathbf{\Phi}}) = \frac{1}{2} \sum_{\boldsymbol{x} \in \Omega} \left(f(\boldsymbol{x}) - g(\widetilde{\mathbf{\Phi}} \cdot \boldsymbol{x}) \right)^2 \quad ?$$

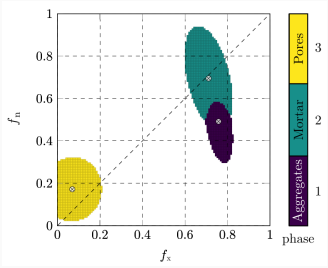
Multimodal registration

A solution

Construct a functional $\mathbf{\Phi}(f_\mathrm{x}, f_\mathrm{n})$ based the joint histogram $p(f_\mathrm{x}, f_\mathrm{n})$.



Joint histogram $p(f_\mathrm{x}, f_\mathrm{n})$

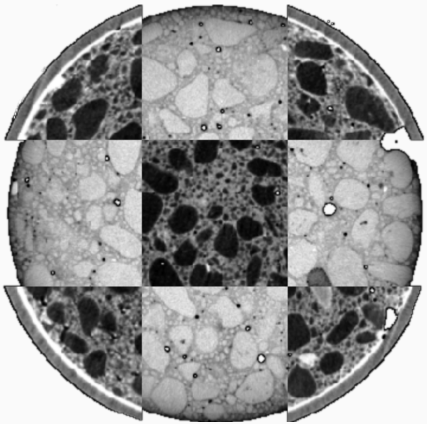


Phase repartition

Tudisco, E., Jailin C., *et al.*: *An extension of digital volume correlation for multimodality image registration*, Measurement Science and Technology (2017)

Roubin E., Andò E., Roux S.: *The colours of concrete as seen by x-rays and neutrons*, Cement and Concrete Composites (2019)

Multimodal registration



MMR in Spam

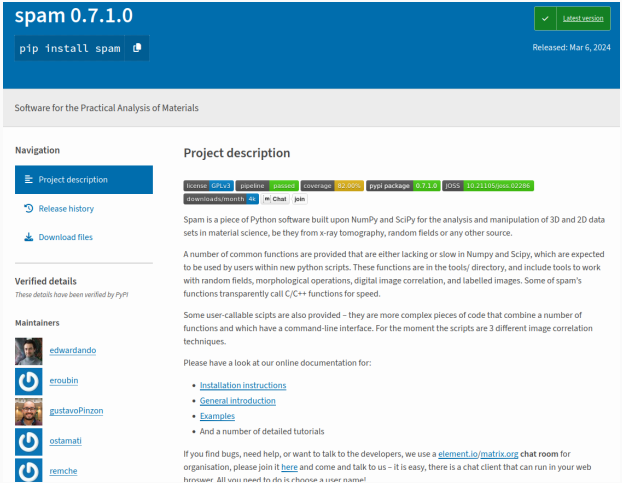
- Online tutorials and examples
- Used in many published works

Going further

- Enhance spatial resolution of neutrons
- Kinematics that map one phase to the next image

General information

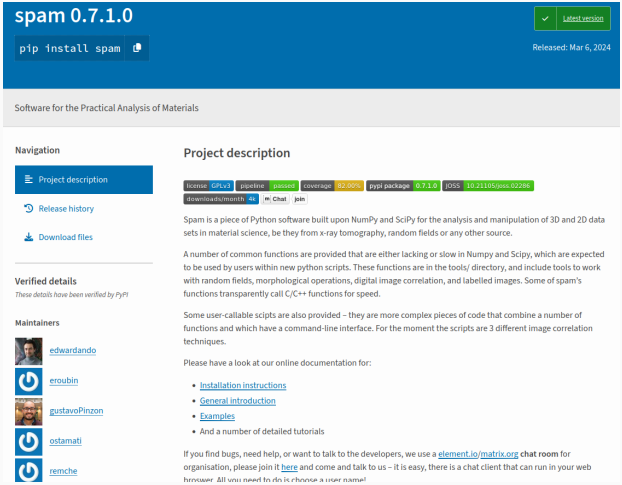
python package

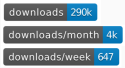


Stamati, Andò, Roubin: *Journal of Open Source Software*, 2020 (Citations: 110)

https://pepy.tech/project/spam

python package





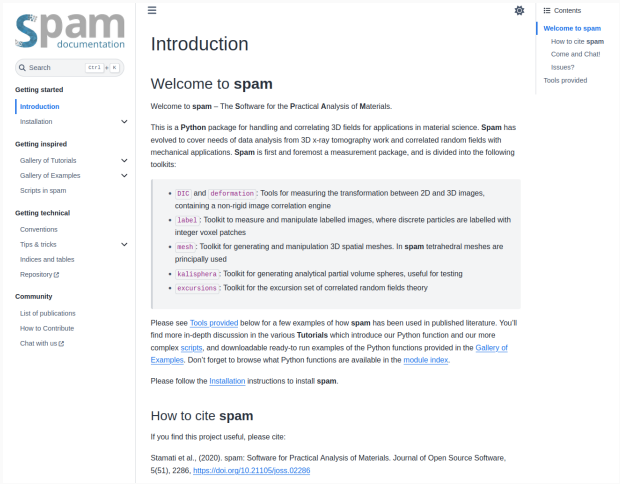
Stamati, Andò, Roubin: *Journal of Open Source Software*, 2020 (Citations: 110)

Easy to use!!

```
pip install spam
```

https://pepy.tech/project/spam

documentation

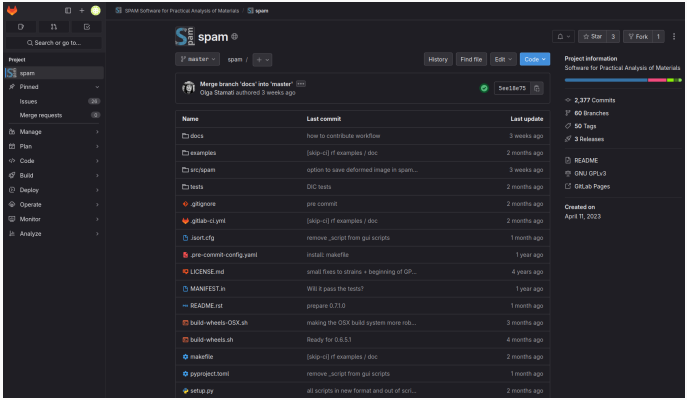


- Installation instructions
- Tutorials
- Examples
- Module indices
- Code repository
- Communication
- How to contribute

Read the Docs

https://www.spam-project.dev/docs/intro.html

repository

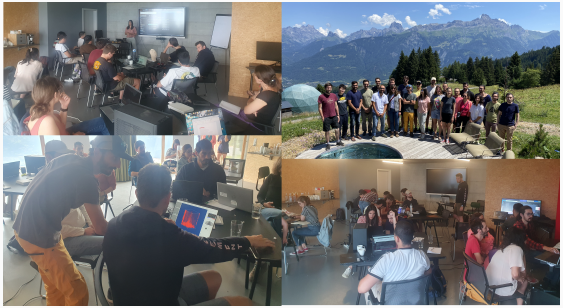


- Source code
- 93% test coverage
- Reporting issues
- Contributing!

`https://gitlab.com/spam-project/spam`

workshops

Valais, Switzerland 18-21/07/2023



https://www.spam-project.dev/workshops/2023/07/

Villar d'Arène, France 6-9/10/2024

https://www.spam-project.dev/workshops/2024/10/

workshops

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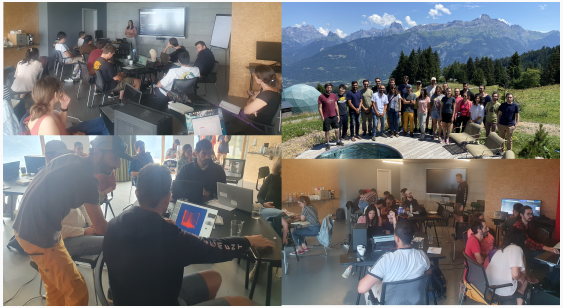

https://www.spam-project.dev/workshops/2023/07/

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https://www.spam-project.dev/workshops/2024/10/

Registrations are open !

People involved

- Core-developers

People involved

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- Contributed with code

...

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- Contributed with code

...

- Contributed with ideas